

Article

The Influence of Improved Strength Grading In Situ on Modelling Timber Strength Properties

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Abstract: The management and preservation of structures in our built environment are central and challenging tasks for practicing engineers. Within the CEN member states (European Committee for Standardization), the so-called Eurocodes form the basis of the design and verification of the load-bearing capacity of structures. Current Eurocodes do not contain special recommendations for existing structures, meaning that the principles for new structures are applied. This can lead to an incorrect estimation of the load-bearing capacity within the semi-probabilistic safety concept. A central task within the investigation and evaluation of existing structures is the strength grading of the material in situ using non-/semi-destructive technical devices. Studies show the potential of the ultrasonic time-of-flight measurement in combination with visual evaluation for an improved grading. The information on the material from an improved grading technique can be used to update the material parameters as a target variable using a measured reference variable. In this contribution, test data from a partner project (spruce, pine, and oak) are analyzed, applying the stochastic grading model of Pöhlmann and Rackwitz. It can be shown that different grading techniques influence the updated distribution function of the material strength within the grade. The results depend on the timber species. Perspectives to develop updated models dependent on the knowledge available are shown and discussed.

Keywords: timber; existing structures; modelling material properties; code calibration; evaluation procedure

1. Introduction

The evaluation of the load-bearing capacity of structural members in existing buildings embraces numerous challenging aspects. At state, there are a few normative regulations, most of which are national codes and guidelines that are not specific for structures made from timber (see, e.g., SIA 269 [1] as a national standard, the German DBV leaflet on concrete [2], and the fib bulletin no. 80 also on concrete [3] just to name a few). However, when dealing with timber, special care is needed due to the natural growth characteristics and high variability of the properties of this material.

To estimate the material strength of a structural member made from timber, the material is graded into strength classes of EN 338:2016-07 [4] applying national grading standards and the assignment criteria of EN 1912:2013-10 [5]. This procedure results in a lower variability of the material properties within a class compared to the ungraded material. The variability of strength properties within a class depends on the quality of the grading procedure, see [6]. The application of grading rules that have been developed for new structures on elements in existing structures is difficult, as

elements are often not fully accessible and not all criteria can be investigated, see, e.g., [7]. Nevertheless, a qualified grading on site enhances the knowledge of the specific element. Depending on the grading procedure, e.g., visual investigation or different nd/sd (non-destructive/semi-destructive) technical devices, the amount of information changes and can be increased by combining different devices. An enhanced knowledge helps to reduce uncertainties concerning the material quality and load-bearing capacity.

An updated material model can be considered within a concept for the standardized verification of the load-bearing capacity of existing timber structures. A suggestion has been developed in [8] that was developed further and applied in a case study in [9]. This is illustrated in Figure 1.

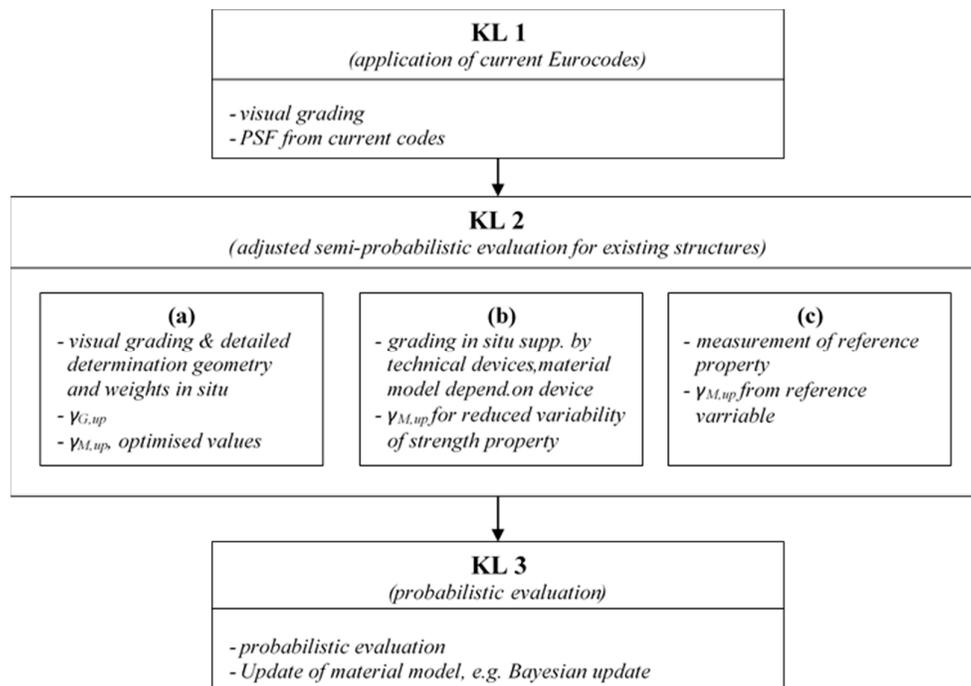


Figure 1. Framework for the evaluation of the load-bearing capacity of existing structures.

The term *Knowledge Level* is based on the JRC Science and Policy Report from 2015 [10]. However, in this concept, this term includes the information available as well as the evaluation format. In this respect, Knowledge Level 1 (KL 1) includes a semi-probabilistic evaluation of the load-bearing capacity, applying partial safety factors (PSF) from current codes without considering any parameter update. What is more, a strength grading is performed, as in most cases in current praxis, visually without advanced technical devices.

Knowledge Level 2 (KL 2) embraces a semi-probabilistic evaluation including different types of parameter update. In Level KL 2a, a more qualitative amount of information such as a good structural performance, freedom of damages or enhanced deformations, etc., are considered. If geometry and permanent loads are investigated carefully in situ, the PSF for permanent actions could be updated, as suggested in SIA 269 [1]. What is more, optimized values for the PSF on the material side are calibrated for adjusted target reliability indexes and chosen design situations. This work is currently under progress. Level KL 2b includes an update of the strength class by grading supported by nd/sd technical devices. The material tests studied in this contribution are part of the work for Knowledge Level 2b, as the influence of the grading procedure on modelling timber strength properties is studied. What is more, in KL 2c, a reference variable measured in situ is used to update the PSF on the material side directly. A formula considering the correlation of target and reference property has been developed in [8], which is described in Section 3.

In Level KL 3, a probabilistic evaluation can be performed. Different statistical tools as, e.g., Bayes updating can be applied to update the random variable based on the knowledge that is

available. A short study on the influence of the prior information and a small number of samples considered within an update is included in this contribution.

In this contribution, the focus lies on level KL 2b. A careful investigation is of the utmost importance to avoid damage to a structure. Thus, the results of calibration tests to analyse the potential of nd/sd grading by an ultrasonic velocity measurement and extraction of core samples in combination with visual grading are presented and evaluated for application in Level 2b. The influence of different grading parameters used on the representation of the material model are studied. To obtain the material distribution function in a strength class, the stochastic grading model by Rackwitz and Pöhlmann [11] is applied. These studies are based on material tests presented in Linke, Rug and Pasternak [12]. What is more, Bayesian Updating is performed to study the influence of the grading procedure applied on the predictive model for a strength parameter exemplary for oak samples. If a reduction of the variability of strength properties by grading supported by technical means in situ can be verified, the partial safety factor (PSF) on the material side (γ_M) could be adjusted.

However, as results show, this cannot be assumed at this state of the research for a grading device in general. An adjustment of the PSF based on a measured reference property is part of Level KL 2c and is illustrated in Section 4.2.4.

2. Materials and Methods

2.1. Test Data

The material tests have been performed at Hochschule für nachhaltige Entwicklung Eberswalde (HNE), within a partner project. Detailed information on grading procedures and timber samples can be found in [12], and the data is collected in [13–15]. By kind permission, the data is used for the studies of this contribution. Table 1 summarizes the scope of the investigation.

Table 1. Grading Parameters and Studied Timber Species-Overview.

Grading Techniques	Visual Grading		
	Direct ultrasonic time-of-flight measurement	Indirect ultrasonic time-of-flight measurement	Density measurement by samples acc. to DIN EN 408:2012-10
Timber species	Oak (301 samples)		
	Spruce (303 samples)		
	Pine (300 samples)		

The specimen have been graded according to visual inspection and the technical devices given in Table 1. As described in Linke, Rug, and Pasternak [12], visual grading underestimated the load bearing capacity. Applying technical different technical devices, the grading yield of a material in a higher strength class could be improved.

Based on ultrasonic velocity and density measurements, Young's modulus has been calculated, see [12] or [13–15]. These values are used as input variable for the reference variable in the stochastic grading model. The corresponding target variable (i.e., bending strength) has been obtained by destructive bending tests, see also [12] or [13–15]. The correlation between the reference and target variable provides insight into the quality of grading based on the nd/sd grading procedures, the results of which are given in Section 3.1

At this stage of the study, visual grading has been combined with one of the grading parameters indicated in Table 1 without considering multiple regressions analysis. This is part of further work.

2.2. The Stochastic Grading Model by Pöhlmann and Rackwitz

A stochastic model to consider the grading procedure for the derivation of a material model can be found in Rackwitz and Pöhlmann [11]. The basic assumptions of the model are explained shortly hereinafter, for details see [11] or [16].

The target variable is Y . By a linear regression model it is connected to the reference variable X , which can be measured directly. This relation can be illustrated as given in Figure 2.

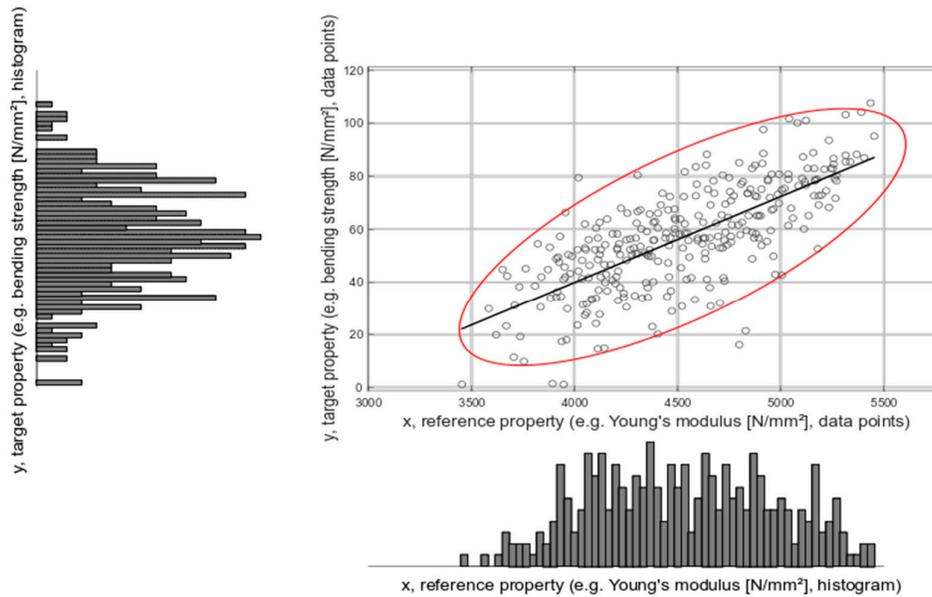


Figure 2. Exemplary scatter plots of reference and target variable with 95% confidence interval error ellipse and linear regression graph.

Within the determination of the parameters of the reference property, errors can occur. Thus, not the reference variable X but a variable Z with an error term τ is measured, so that

$$Z = X + \tau \quad (1)$$

where X is the measured reference variable and τ is the normally distributed error term with $\tau \sim N(0, \sigma_\tau^2)$. The target variable Y is not dependent on the measurement error τ .

It is assumed that the measured variable X is normally distributed with $N(\mu_E, \sigma_E^2)$. The probability density function (PDF) of the target variable Y within a certain class is derived by Pöhlmann and Rackwitz [11] as follows:

$$f_y(y) = \frac{1}{K} \frac{\sigma}{\sigma_\epsilon \sigma_E} \varphi\left(\frac{y-a}{b} - \mu_E\right) / \sigma_M \left(\Phi\left(\frac{C_o}{\sqrt{1+C_l^2}}\right) - \Phi\left(\frac{C_u}{\sqrt{1+C_l^2}}\right) \right) \quad (2)$$

where K is a normalizing constant obtained from Equation (10), σ can be calculated by applying Equation (3), μ_E is the expected value, σ_E is the standard deviation of the normally distributed measured variable X , σ_ϵ is the standard deviation on the normally distributed error term, and a and b are the parameters of the linear regression, see Equation (9). The variables σ_M , C_o , C_u , and C_l and y can be obtained from Equations (4)–(9).

$$\sigma = \frac{\sigma_\epsilon \sigma_E}{b \sigma_M} \quad (3)$$

$$\sigma_M^2 = \left(\frac{\sigma_\epsilon^2}{b^2} + \sigma_E^2 \right) \quad (4)$$

$$m = \frac{(y-a)/(\sigma_E^2/b) + \mu_E/\sigma_E^2}{1/(\sigma_E^2/b^2) + 1/\sigma_E^2} \quad (5)$$

$$C_o = \frac{g_o - m}{\sigma_\tau} \quad (6)$$

$$C_u = \frac{g_u - m}{\sigma_\tau} \quad (7)$$

$$C_l = \frac{\sigma}{\sigma_\tau} \quad (8)$$

$$y = a + bx + \varepsilon > 0 \quad (9)$$

The normalizing constant K can be calculated

$$K = \Phi\left(-\frac{C'_o}{\sqrt{1 + C_1'^2}}\right) - \Phi\left(-\frac{C'_u}{\sqrt{1 + C_1'^2}}\right) \quad (10)$$

with

$$C'_o = \frac{g_o - \mu_E}{\sigma_\tau} \quad (11)$$

$$C'_u = \frac{g_u - \mu_E}{\sigma_\tau} \quad (12)$$

$$C'_l = \frac{\sigma_E}{\sigma_\tau} \quad (13)$$

where g_o and g_u are limiting values of the grading parameter for the certain class, μ_E is the expected value of the normally distributed and measured variable X , and σ_τ is the standard deviation of the normally distributed error τ .

The cumulative distribution function (CDF) can be obtained from Equation (2) by numeric integration [16]. Applying this grading model, the correlation between a reference variable X and target variable Y as well as grading regulations are considered to develop the material model within a certain class. As the concept is based on a two-dimensional normal distribution, the probability density function within the strength class from Equation (2) corresponds to a normal distribution. However, in reliability analyses, strength properties are modelled by lognormal distributions to avoid negative values. The parameters of the corresponding lognormal distribution can be calculated from the parameters of a normal distribution easily.

3. Results—The Material Model Applying the Stochastic Grading Model

3.1. General Remarks

This section shows the results of this study, applying the stochastic grading model dependent on the timber species. Applying the stochastic grading model, the probability density function of the target property (bending strength) in a strength class, by considering the correlation between the reference and target variable, is calculated.

Technical devices have been checked according to DIN 4074-3:2008-12 [17]. Based on this, the error term in the stochastic model can be assumed to be normally distributed with $\tau \sim N(0, 20^2)$ [m/s] for ultrasonic measurements. The value is a mean value for all timber species investigated. For density measurements, the error term consists of the weight and the geometry measurement, and can be taken $\tau \sim N(0, (1.76)^2)$ [kg/m³] for spruce and pine and $\tau \sim N(0, (2.70)^2)$ [kg/m³] for oak for these very sensible devices (0.4% of the expected value of the density determined from tests). Limiting values for the stochastic grading have been applied, as recommended in [12]. The following sections show the results. Characteristic values are given as 5%-Quantile of a Lognormal distribution. The results are discussed in Section 4, and the results of an exemplary Bayesian update using the material model developed in this section are illustrated.

3.2. Oak Samples

The results for grading using the ultrasonic device are given in Table 2 and are illustrated in Figure 3 for grading by an indirect ultrasonic time-of-flight measurement. As the correlation between the density measurement and the bending strength of oak samples was too low, no reliable results can be shown here.

Table 2. Results for oak samples and different grading techniques obtained from the stochastic grading model.

Strength Class	Direct US Measurement			Indirect US Measurement			Density Measurement ²		
	ρ	$COVR$ [-]	f_k [N/mm ²]	ρ	$COVR$ [-]	f_k [N/mm ²]	ρ	$COVR$ [-]	f_k [N/mm ²]
D30	0.69	0.26	31.17	0.72	0.28	27.05	-	No sufficient correlation!	
D35		0.24	35.57		0.25	33.42			
D40		0.22	41.57		0.23	38.41			
>D40		0.17	60.30		0.16	61.62			

¹ Limiting values for stochastic grading: [12]. ² Limiting values for stochastic grading: 5%—quantiles from EN 338

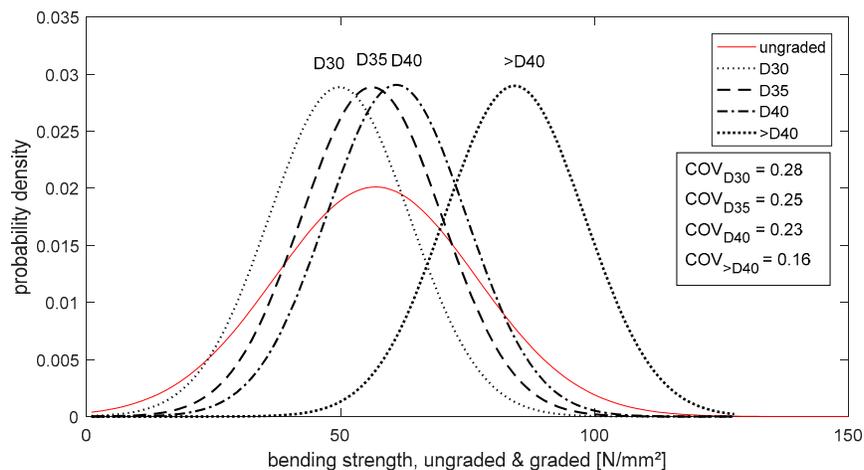


Figure 3. Probability density functions of graded and ungraded materials from the stochastic grading model for oak samples and strength grading based on an indirect ultrasonic-time-of-flight measurement.

The correlation coefficient of bending strength and Young's modulus calculated from direct and indirect ultrasonic measurements are strong for a single grading parameter ($\rho = 0.69$ and $\rho = 0.71$). For comparative values see, e.g., [7] (correlation coefficients of indicating property and destructively measured strength property determined from coefficient of determination for knots $p = \sqrt{0.15} \dots \sqrt{0.35} = 0.39 \dots 0.59$ and for density $p = \sqrt{0.20} \dots \sqrt{0.40} = 0.45 \dots 0.63$). What is more, the results gained from grading the oak samples show a low variability of the bending strength within the classes D40 and better than D40 when graded by ultrasonic time-of-flight measurements. Characteristic values are similar to the values in EN 338:2016-07 [4] or even higher. Thus, visual grading supported by ultrasonic measurement seems to be well suited to supporting the grading procedures of structural oak members.

The reason for the correlation coefficient of density and bending strength being very low in these tests remains to be studied.

3.3. Spruce Samples

The results for spruce samples are summarized in Table 3. Figure 4 illustrates the probability density functions of the bending strength of the ungraded and graded material exemplary for an indirect ultrasonic time-of-flight measurement.

Table 3. Results for spruce samples and different grading techniques obtained from the stochastic grading model.

Strength Class	Direct US Measurement		Indirect US Measurement			Density Measurement ²			
	ρ	$COVR$ [-]	f_k [N/mm ²]	ρ	$COVR$ [-]	f_k [N/mm ²]	$COVR$ [-]	f_k [N/mm ²]	
C18	0.42	0.42	11.26	0.44	0.41	11.39	0.30	0.42	17.76
C24		0.36	17.27		0.36	17.05		0.39	19.78
C30		0.32	22.03		0.32	21.65		0.38	21.12
>C30		0.29	27.82		0.28	28.21		0.34	26.14

¹ Limiting values for stochastic grading: [12]. ² Limiting values for stochastic grading: 5%—quantiles from EN 338

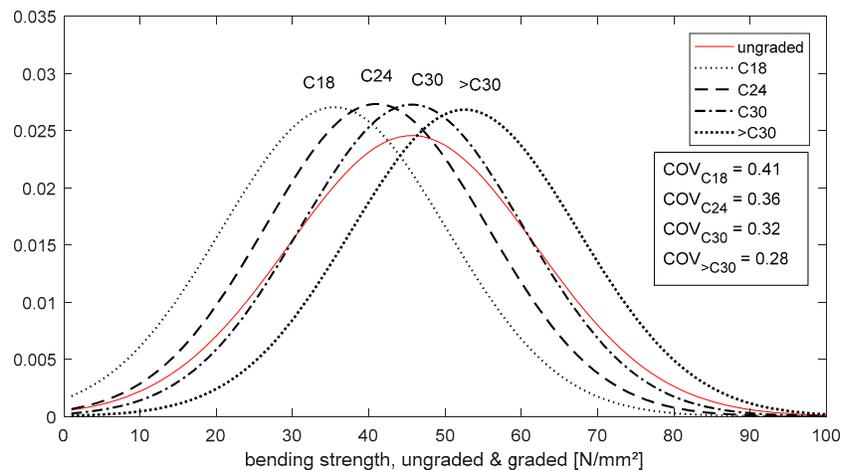


Figure 4. Probability density functions of graded and ungraded materials from the stochastic grading model for spruce samples and strength grading based on an indirect ultrasonic time-of-flight measurement.

The correlation coefficient of bending strength and Young's modulus calculated from direct and indirect ultrasonic measurement and also for density measurements are moderate ($\rho = 0.42$ and $\rho = 0.44$, $\rho = 0.30$). Table 3 and Figure 4 show higher coefficients of variation (cov) of the strength property compared to the studies on oak members, which is probably due to the comparable lower correlation coefficients. Besides, the characteristic values (a 5% quantile based on a lognormal distribution) are relatively low compared to the values given in EN 338:2017-07 [4]. However, similar to the studies on oak samples, the coefficient of variation of the strength property in the class is reduced for strength classes greater than C24 (greater than D35 for oak) compared to the ungraded material.

For single grading parameters, the results are promising. It can be assumed that, by a combination of more parameters, even better results can be achieved.

3.4. Pine Samples

Studies on pine samples showed correlations of $\rho_{US,dir} = 0.23$ for direct and $\rho_{US,indir} = 0.27$ for indirect ultrasonic time-of-flight measurements. For grading based on density measurements on small clear samples, the correlation coefficient is $\rho_{dens} = 0.54$, which is relatively high compared to the other samples studied here.

As almost all samples have been graded to strength class C40, no different results for strength classes can be shown. The coefficient of variation is $cov = 0.36$, the expected value is $\mu = 53.72$. Thus, solely visual grading seems to underestimate the load-bearing capacity. However, this great difference among different softwood species, needs to be considered in further developments, see also [12].

4. Discussion

4.1. Evaluation of Results

It can be concluded that the quality of the grading procedure based on different technical devices depends on the timber species. The great potential of the ultrasonic time-of-flight measurement as a grading parameter can be shown for oak samples. The variability of strength parameters in the classes were low, characteristic values (a 5% quantile) were high. For spruce and, especially, pine the correlation of Ultrasonic Measurements (USM) with the strength properties as a single grading parameter were low. This is probably due to timber species specific properties as, e.g., a high KAR (knot area ratio) value.

At first sight, these correlation values seem to be low. However, it has to be emphasized that the grading parameters have been analyzed independently. The load-bearing capacity of timber as an inhomogeneous material depends on a range of parameters, which have to be considered jointly. Being focused on single parameters, the results are promising. For future work, the multiple correlation of grading parameters has to be analyzed. This leads to an accounted reduction in the variability of material properties within the classes. Further work on this will be presented.

The following sections collect ideas on how to use updated information within the evaluation of the load-bearing capacity of members in existing structures and options to consider information from an improved grading and measured reference variable within the evaluation.

4.2. Options to Consider Updated Information within the Evaluation of Load-Bearing Capacities

4.2.1. General Idea

When considering the grading procedures applicable for elements in existing structures, the challenge is to find options to take into account updated information within the evaluation of the load-bearing capacity.

One idea is to develop a new prior model that constitutes of a combination of visual grading and different technical devices. This model could then be used as a basis for an adjustment of the PSF depending on the amount of information collected in situ and for a Bayesian update of the material model. Based on this information, the posterior and the predictive model can be developed using additional test data from a specific object. For an illustration of the procedure, see Figure 5. The levels are explained shortly in Figure 1.

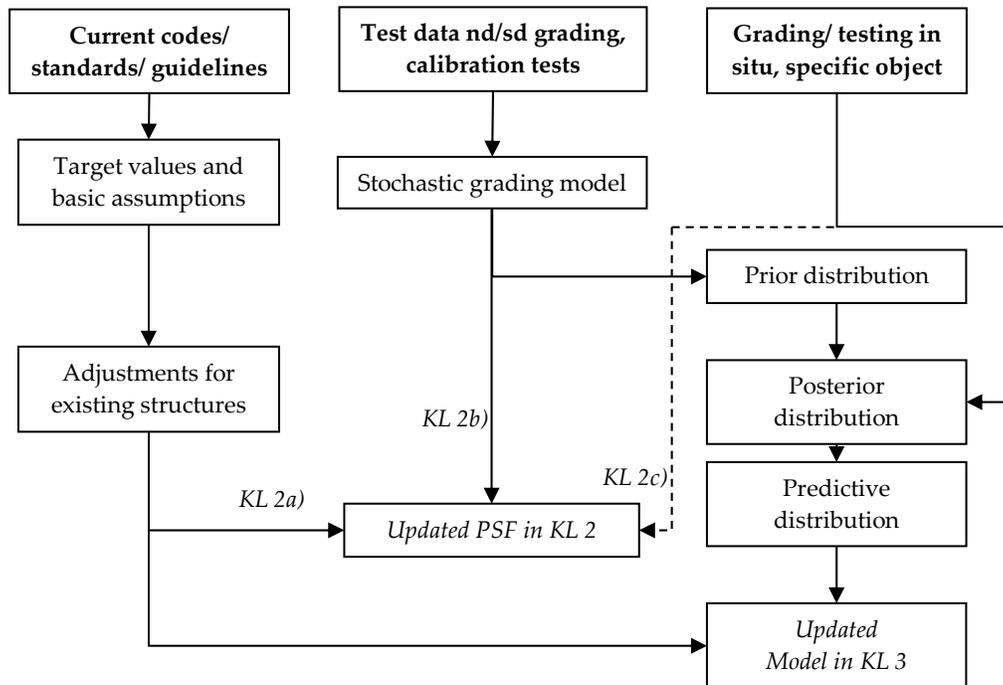


Figure 5. Considering different sources of information.

If the material is graded by visual inspection without actually measuring specific properties, the material model may be developed from EN 338 and the Probabilistic Model Code (PMC) by the Joint Committee on Structural Safety (JCSS) [18]. This model may be used as a prior model for a Bayesian update and as the basis to derive a target safety level for structures designed by current regulations. For specific grading procedures, the material model for prior distribution and for an adaption of safety factors may be adjusted. As mentioned above, a model including multiple correlations between grading parameters has to be developed to reduce the variability of parameters, and work is still under progress. However, perspectives shall be shown and discussed.

4.2.2. Application on Test Data

For illustrating purposes, it is assumed that a fictive structure is investigated. The material is identified to be oak and graded by a combination of visual grading and indirect USM to strength class D40. What is more, five samples could be taken, these lead to estimated values for the bending strength of $f_{m,ex,n} = [61.8 \ 80.5 \ 79.5 \ 55.1 \ 85.1]$ N/mm². For this example, five of the samples tested in [5] that have been graded by visual inspection and indirect ultrasonic time-of-flight by one of the criteria of D40 and by the other criteria into a higher class, have been chosen randomly (samples Ei-7-103, Ei-7-112, Ei-7-128, Ei-7-129, Ei-7-182) to generate realistic values.

Based on Table 2, the *cov* for the bending strength in class D40 (oak samples) and the grading by indirect USM is taken $cov_{m, D40} = 0.22$. The characteristic value is not taken from the calibration test but from EN 338:2016-07 [1] $R_{k, D40} = 40$ N/mm².

4.2.3. Bayesian Update of the Material Model (KL 3)

Based on a Bayesian estimation, the material model is updated by a joint consideration of prior and additional information. The posterior model is developed as follows:

$$f_{\theta}''(\theta|\hat{x}) = \frac{f_{\theta}'(\theta)L(\theta|\hat{x})}{\int f_{\theta}'(\theta)L(\theta|\hat{x})d\theta} \quad (14)$$

where $f'_\theta(\theta)$ is the probability density function of a random variable based on prior information, $L(\theta|\hat{x})$ is the likelihood, and $\int f'_\theta(\theta)L(\theta|\hat{x})d\theta$ is a normalizing factor. With the posterior probability density function, the predictive function $f''(x)$ can be calculated [19] as follows:

$$f'''(x) = \int fx(x|\theta)f''_\theta(\theta|\hat{x})d\theta \quad (15)$$

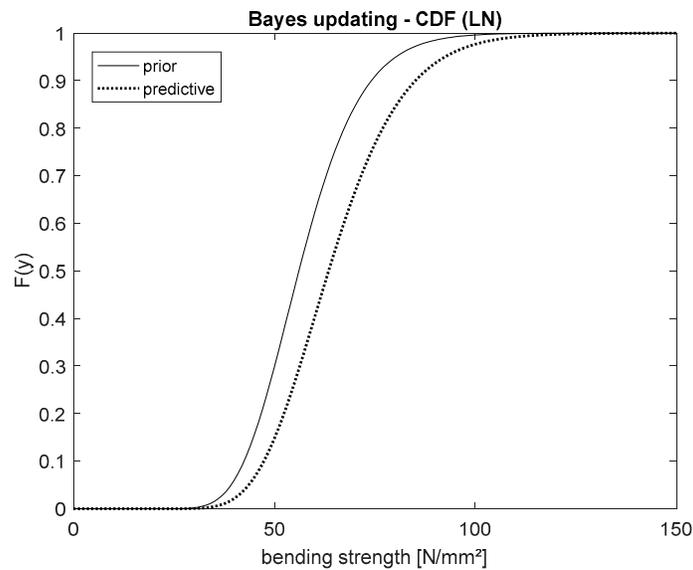
where $f''_\theta(\theta|\hat{x})$ is the posterior probability density and $fx(x|\theta)$ is the probability density function of the x dependent on θ . The integrals can be solved numerically or by simulation. For normal distributions, analytical solutions exist, see, e.g., [19].

For this contribution, the analytical procedure to obtain the predictive model described in [19] is applied. As prior and posterior distribution functions can be assumed to belong to the same distribution type, the prior distribution is a conjugate prior. To consider the trust of the engineer in the data, the uncertainty of information is considered within the updating procedure.

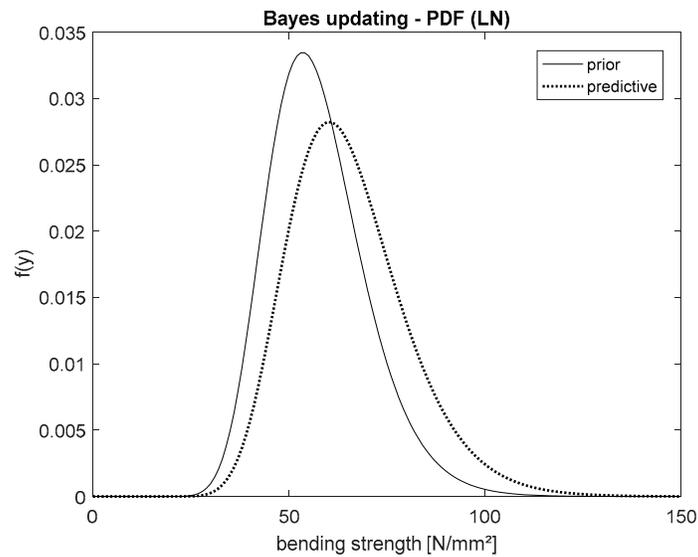
The cumulative distribution function (CDF) and the probability density function (PDF) are illustrated in Figure 6a and Figure 6b, respectively, and the parameters are given in Table 4.

Table 4. Exemplary Bayesian updating of bending strength based on the oak samples–prior model from the calibration test.

	m [N/mm ²]	cov [-]	x _k [N/mm ²]	Notes
Prior	57.44	0.22	40	cov: Result of calibration tests for indirect USM (oak) m and R _k : from EN 338 D40
Data	72.38	0.18	53.53	Five randomly chosen samples from database
Predictive	65.17	0.23	44.61	



(a)



(b)

Figure 6. Bayes updating: (a) the cumulative distribution function; (b) the probability density function

The expected value of the predictive material model is higher than that of the prior model. The coefficient of variation is also slightly higher, as within the updating procedure the uncertainties resulting from the original (prior) model and the test results are coupled. The predictive distribution function may be used to verify the load-bearing capacity of the member within a probabilistic evaluation (KL 3).

Using Bayes updating to include prior information on the material model in a strength class as prior information and then updating it based on tests seems to be a promising approach. However, it has to be emphasized that the influence of the prior distribution on the predictive model is quite high. Thus, a careful choice of this model is important. To develop a statistically reliable prior model for different grading devices, extensive testing for different timber species have to be carried out. What is more, the influence of the combination of different grading parameters on the prior model have to be studied. As for this contribution, the assumptions for the prior model are based on a calibration test without a multiple regression and results that were only satisfying for oak, and the concept is still under development. Thus, at state, it is recommended to use assumptions from the Joint Committee on Structural Safety (JCSS) Probabilistic Model Code (PMC) and EN 338 to establish a prior model and apply results from calibration tests or measurements on site as posterior information to update the model for special cases. Applying this on the example shown above this would alter the prior model and thus the predictive model, as given in Table 5.

Table 5. Exemplary Bayes updating of bending strength based on an oak samples–prior model from the Probabilistic Model Code (PMC).

Distr.	m [N/mm ²]	cov [-]	x _k [N/mm ²]	Notes
Prior	60.45	0.25	40	cov: JCSS PMC [18] R _k : from EN 338 D40
Data	72.38	0.18	53.53	Five randomly chosen samples from database
Predictive	66.19	0.26	41.95	

This model is slightly more conservative as the cov is higher and the characteristic value is lower compared to Table 4. However, there are different things influencing the predictive model. For example, the weighting of the prior model also has an influence.

As uncertainties of prior and data are coupled, the cov of the predictive model cannot be lower than the cov of the prior in this approach, even if the data show a low value as in this example. Thus, a reduction of the PSF cannot be realized in that way. Thus, this model is more suitable for a probabilistic evaluation considering updated information.

However, an option to consider updated information within the update of the PSF considering an improved expected value of the target property has been developed in [8] and is presented in Section 4.2.4.

4.2.4. Update of the Partial Safety Factor Based on Testing (KL 2c)

It is not always possible to extract samples and evaluate them in destructive tests, but it is always possible to measure reference properties. Information from reference variables measured with nd/sd technical devices can also be used to update information on a certain target variable. Principles have been described in [20] as background information on SIA 269:2011 [1]. These principles are used to develop a formula to update the PSF to be applied on a certain material resistance based on the measurements of a reference variable. The formula has been developed and published in [4], the development is described hereinafter with respect to the mentioned reference. First, the mean value of the target variable depending on the measurement $\mu_{y|x_{meas}}$ is calculated as follows:

$$\mu_{y|x_{meas}} = \mu_{y_{code}} \cdot \left(1 + \rho_{x,y} \cdot cov_{y_{code}} \cdot \frac{x_{meas} - \mu_{x_{code}}}{\mu_{x_{code}} \cdot cov_{x_{code}}} \right) \quad (16)$$

where $cov_{y_{code}}$ is the coefficient of variation of the target variable as defined in the code, $cov_{x_{code}}$ is the cov of the measured variable as defined in code, $\mu_{x_{code}}$ is the mean value of measured variable as defined in code, $\rho_{x,y}$ is the correlation coefficient of the target variable, and the measured variable x_{meas} is observed by a nd/sd test in situ, see also [20]. The standard deviation of the target variable $\sigma_{y|x_{meas}}$ depending on the measurement is as follows:

$$\sigma_{y|x_{meas}} = cov_{y_{code}} \cdot \mu_{y_{code}} \cdot \sqrt{1 - \rho_{x,y}^2} \quad (17)$$

With Equations (1) and (2), the cov of the target variable $cov_{y|x_{meas}}$ depending on the measurement can be calculated as follows:

$$cov_{y|x_{meas}} = \frac{\sigma_{y|x_{meas}}}{\mu_{y|x_{meas}}} \quad (18)$$

$$V_{y|x_{meas}} = \frac{cov_{y_{code}} \cdot \sqrt{1 - \rho_{x,y}^2}}{\left(1 + \rho_{x,y} \cdot cov_{y_{code}} \cdot \frac{x_{meas} - \mu_{x_{code}}}{\mu_{x_{code}} \cdot cov_{x_{code}}} \right)}$$

The PSF γ_m can be calculated for lognormal distributed variables as follows:

$$\gamma_m = \exp\left(cov_R \cdot (\alpha_R \cdot \beta + \Phi^{-1}(q))\right) \quad (19)$$

The determination of the PSF according to Equation (4) is, in general, referred to as the Design Value Method where α_R is the so-called sensitivity factor and β is the target reliability for a 50-year reference period. This method is also described in ISO 2394:2015 [21]. With $cov_R = cov_{y|x_{meas}}$, the updated PSF can be calculated as follows:

$$\gamma_{M,up} = \exp\left(\frac{cov_{y,target} \cdot \sqrt{1 - \rho_{x,y}^2}}{1 + \rho_{x,c} \cdot cov_{y,target} \cdot \frac{x_{meas} - \mu_{x,ref}}{\mu_{x,ref} \cdot cov_{x,ref}}} \cdot (\alpha_R \cdot \beta + \Phi^{-1}(q))\right) \quad (20)$$

The update of the PSF depends on the resistance variable that is now conditional on the observed nd/sd test in situ, x_{meas} . To include a model uncertainty the model factor γ_{Rd} is considered and the PSF $\gamma_{M,up}$ is calculated using the following equation:

$$\gamma_{M,up} = \gamma_{Rd} \cdot \gamma_{M,up} \quad (21)$$

Again, as in Section 4.2.3, it is assumed for exemplary purposes that the timber in a structure is graded by a combination of visual inspection and indirect USM. To class D40, the material is identified as oak. However, in contrast to Section 4.2.3, no extraction of samples is possible. It is clear that, also in this case, a probabilistic evaluation based on measured reference properties could be performed. However, for these contribution, this constructed scenario shall be used to apply the principle updating the PSF described above.

To update the PSF using this principle, a target reliability index or probability of failure has to be defined. The determination of a target reliability level is a complex topic with multiple issues to be considered. The level of information when erecting new structures and rehabilitating existing ones is fundamentally different, as the latter already exists in tangible form, can be investigated, and its structural performance can be considered. Thus, different adjustments of target values for existing structures are discussed in the literature, see, e.g., [22] or [23]. Here, $\beta = 3.2$ is assumed for a reference period of $T_{ref} = 50a$.

With $\rho_R = 0.8$ [24] and $q = 0.05$, the second part of Equation (20) becomes 0.915. The correlation coefficient is taken from Table 2 $\rho_{x,y} = 0.72$, the measured variable is $x_{meas} = 4.764 \times 10^3$ m/s (mean value of measurements), $\mu_{x,ref}$ is taken as $\mu_{x,ref} = 4.602 \cdot 10^3$ m/s for D40 from [25] as this is the boundary to grade a member into this class. The coefficient of variation of the measured variable $COV_{x,ref}$ is taken from the JCSS Probabilistic Model Code [10], $COV_{x,ref} = 0.10$, and the coefficient of variation of the target variable $COV_{y,target}$ is taken from the prior distribution (Table 4)) $COV_{y,target} = 0.22$.

The model factor is derived by assuming a normal distribution for the model uncertainty and the adjustment for an accompanying variable. It is taken that $\gamma_{R,d} = 1.08$ (see [4]). Applying these values, $\gamma_{M,up} = 1.24$ is calculated for this example.

At state, the PSF for solid timber is $\gamma_M = 1.30$. The update of the factor leads to a reduction, which is due to the fact that the measured reference variable is higher than the value given in the code for this class. The remaining potential can then be used by this update to activate load-bearing reserves. If the calculation gives a higher safety factor, one should consider grading the material into a lower strength class instead of applying a higher safety factor. An update of the PSF for different measured reference variables have been presented in [26].

5. Summary

This contribution analyses options to improve the material model based on the nd/sd grading of timber elements in existing structures. First, the stochastic grading model of Pöhlmann and Rackwitz [11] is applied. The results show that the correlation coefficients for an ultrasonic time-of-flight measurement and bending strength depend on the timber species. Evaluated as single grading parameters, the correlation has found to be low for pine, moderate for spruce, and very good for oak. The correlation of the bending strength with density measured on small clear samples as a single grading parameter has been found to be moderate. These results comply with results given in [12]. These different values result from the numerous parameters that influence the load-bearing capacity of a member made from natural grown timber. It becomes clear that having multiple regression coefficients of different grading parameters could help to consider more grading parameters simultaneously. Combining the information gained from visual inspection, ultrasonic measurements, and the extraction of core samples for density measurements increases knowledge and hence reduces uncertainties concerning the load-bearing capacity.

What is more, options to combine prior and updated information are studied. In this respect, the material model generated by the stochastic grading model is used as prior information that can be updated by Bayesian updating. Here, the influence of the prior distribution function on the updated model has to be emphasized. Thus, a careful choice of this model is of great importance. What is more, the variability of the material model cannot be reduced by this procedure as uncertainties from calibration test and the in situ testing are coupled.

Within further work, the idea to develop new prior models depending on the grading devices should be extended and studied further. Another major part of this is the calibration of adjusted PSF

in level KL 2a. Here, optimization potential can be generated by calibrating PSF for different design situations.

The development of tools and concepts for a careful investigation and rehabilitation of existing structures is of utmost importance for the building industry. The preservation of existing timber structures not only saves cultural heritage, but also helps to avoid waste and unnecessary consumption of resources and energy.

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